PAPER

2D Feature Space for Snow Particle Classification into Snowflake and Graupel

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SUMMARY This study presents three image processing systems for snow particle classification into snowflake and graupel. All of them are based on feature classification, yet as a novelty in all cases multiple features are exploited. Additionally, each of them is characterized by a different data flow. In order to compare the performances, we not only consider various features, but also suggest different classifiers.

The best achieved results are for the snowflake discrimination method applied before statistical classifier, as the correct classification ratio in this case reaches 94%. In other cases the best results are around 88%.
key words: *image processing, snow particle features, classification*

1. Introduction

The Japan coast of the Japan Sea is characterized by the orographic snowfall. Clouds are created over the sea and then transported along the direction of a prevailing winter monsoon wind (northwestern direction). The heavy snowfall is explained [5] as a consequence of the topographic updraft, which causes snow clouds to develop. Additionally, the stronger the wind, the greater the riming growth of particles is. It has also been shown by the statistical calculation over the period of 16 years data from many meteorological station around Japan [10] that graupel precipitation is predominant in this region. Besides, it has been also noticed that snowflake size distribution varies in this area [7], as its creation is influenced by the temperature, place, and particle density. Similarly, it has been proved [4] that when the temperature distribution changes within the cloud, the internal structure of graupels changes as well.

On the other hand, the ground observation of snow particles plays an important part in broad understanding of the processes responsible for the snow cloud growth, as well as a singular particle physics behaviour. They allow to improve the information acquired in lower atmosphere by radars and lidars [14]. For instance, it has already been verified [6] that the relation of the reflectivity factor (Z) and the precipitation rate (R) does not depend on the snowfall intensity, as it was suggested by Marshall and Gunn [9]. Besides it was proven that the average size distributions of snowflakes maintain their exponential form with increase in snowfall intensity and that there occur small changes in the slope of the size distribution even in the condition of an equal snowfall intensity. Finally, it is also known that the characteristics are different for snowflake and graupel, which has been exploited in the research for graupel and hail recognition using multiparameter radar [2].

Therefore, we argue that an acquisition of snow particle images and its classification into graupel and snowflake could be exploited for participation parameter calculation. Moreover, this local information can be further utilized for more detailed snowfall phenomena description which could be useful to explain e.g. the local changes in snowflake distribution described in [7].

In this paper we present three developed systems for snow particle classification into snowflake and graupel. All of them exploit at least two features which describe a snow particle. There have already been suggested shape and statistical parameters or some specially designed features which take into consideration the characteristics of the particles [15]. The emphasis on this research was to find such features which give high accuracy classification. Additionally, the novel procedure for classification is described and the classification efficiency of this and statistical classifiers is compared. We believe that the described techniques become a powerful tool for local snowfall parameter calculation.

2. Imaging system overview

In order to capture the images of falling snow particles in sufficient resolution of 1280 x 960 the camera with shutter speed of 1/10000 s is exploited. It is connected to the computer via the IEEE 1394b interface. Enlightening measuring volume (128W x 96H x 250D mm) by four halogen lamps improves the image quality as well as enables the system to work during the night. The camera is placed in the distance of two meters from the measuring volume inside a vertical tunnel to protect the equipment from the weather influence. Moreover,

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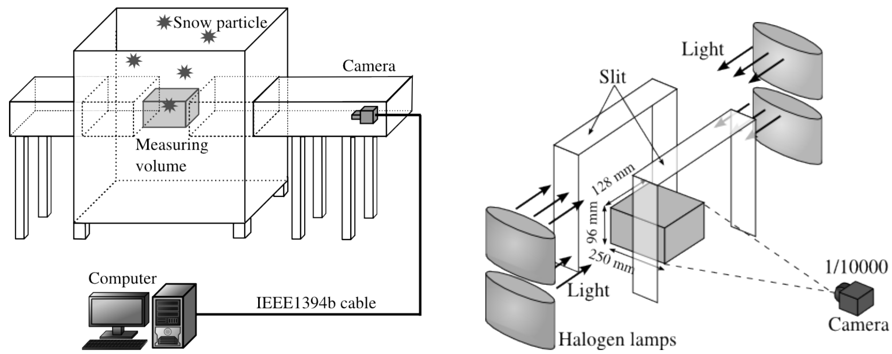


(a) General (b) Measuring volume

Fig. 1 The system overview.

wind breakers are installed around (see Fig. 1).

The measurement took place at the Kanazawa University in the winter months of January and February, 2009. A labelling method extracted snow particles from gathered data. Furthermore, the visual inspection of the particle database allowed to enrich it with information to which class belongs each particle.

3. Snow particle shape features

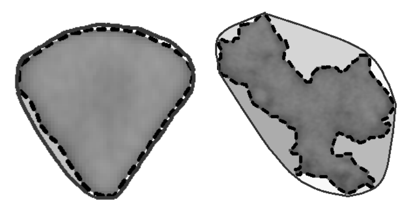
There are many features [15] introduced for classification of the snow particles into snowflakes and graupels. Its definitions exploit various kind of information encoded in image. Some features base on general knowledge about snow particle surface characteristics and its falling behaviour. Other utilize the contour complexity to derive the feature value or use the illumination changes as a main factor for particle description. Finally, there are also described those from shape and statistical parameters, which proved to be effective for automatic snow particle classification.

3.1 Area size

For images with normalized resolution it is possible to compare object area size. It was noticed that the homogoneous graupel structure has larger area than snowflake. Although, the snowflake may seem bigger due to its branching structure.

3.2 Perimeter

It has been noticed from the visual inspection that the contour of a snowflake is longer than that of a graupel. This is because of the complicated snowflake object structure. Therefore, the perimeter of the particle becomes a feature. In this research this feature is calculated for original resolution images as well as for normalized ones (images scaled to the 40 x 40 pixels resolution).



(a) Graupel (b) Snowflake

Fig. 2 The idea of concave number feature. Dashed line represents contour; solid line the convex hull; different colours mark the concave parts.

3.3 Concave number

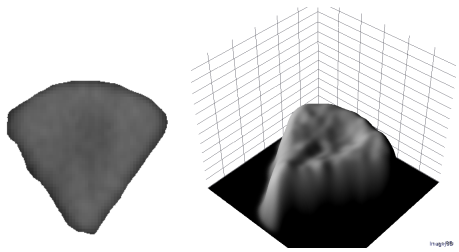
The snowflake contour is not only longer but also more complex than that of graupel. Looking from the outside at snowflake particle shape, it is noticeable how many concave parts are in the contour. In order to calculate its number, the convex hull for the object is created. Then, each time the contour splits from the convex hull line, a new concavity is marked and its number is counted. Fig. 2 presents an example of this feature calculation. Each concavity is marked with different colour.

3.4 Brightness

Due to describe globally the changes in illumination in the image, the *brightness* feature has been defined. Actually, this parameter may be calculated in many ways. However, in our research it has been noticed, that the surface illumination of graupel changes slowly in the comparison of snowflake. Therefore, this feature is defined as the maximal difference of illumination within 3 x 3 pixel window, which passes through the object.

3.5 Volume

When the snow particle image is taken the parts of particle which are closer to the camera are brighter than those further. Therefore, we can read the object illumination values as a transformed volume of the object



(a) Original image (b) Volume representation

Fig. 3 Visualization of volume feature idea.

(see Fig. 3). Hence, in order to calculate the volume all intensity values are summed up. Additionally, due to remove the influence of the size of the particle the result is divided by the object area size, see the equation:

$$V = \frac{\sum I(x,y)}{Area} \tag{1}$$

where $I(x,y)$ is the intensity of the image higher than the background threshold.

3.6 Roughness

The *roughness* feature describes the relation between the object perimeter and its area size [12]. On the other hand, it could be seen as a proportion of the perimeter of the object with the perimeter of the circle of the same area size. It is given with the formula:

$$roughness = \frac{Perimeter}{2\sqrt{Area * \pi}} \tag{2}$$

3.7 Danielsson

The relationship between object area and minimal distances from each object point to the contour are described by *Danielsson* feature [13]. In order to calculate its value following formula should be applied:

$$Danielsson = \frac{Area^3}{\sum_j min_i(dist(object_i, contour_j))} \tag{3}$$

where j iterates through contour points, i iterates through object points, the *dist* function calculates Euclidean distance between given points and *min* returns the minimal value.

3.8 Hu moment 1st order

The Hu moment of the 1st order describes image feature basing on image illumination and its position in image [8]. The parameter, therefore, has been calculated for original and thresholded images.

4. Classification methods

In order to classify snow particles *Mahalanobis minimum distance* and *k-nearest neighbourhood* classifiers has been chosen. Additionally, a *double threshold* classifier is introduced.

4.1 Mahalanobis minimum distance classifier (MMD)

The definition of *Mahalanobis minimum distance classifier* [1], [11] takes its origin from the Bayes theorem. It assumes the normal distribution of the likelihood, moreover the covariance matrices should be similar for each class and non-diagonal. It allows to simplify the quadratic discriminant function equation into:

$$g_i(A) = -\frac{1}{2}(A-\mu_i)^T \sum{}^{-1}(A-\mu_i) \tag{4}$$

where i is a class index, A is a feature value, $\sum^{-1}$ is an inverse of covariance matrix $\sum$, and μ_i is the class average value of the feature. In other words, this equation describes a hyper-ellipsoid which encloses inside all values representing one class. When hyper-ellipsoids for different class overlap, then in this area it is difficult to classify. On the other hand, when object is placed in the area which does not belong to any of the hyper-ellipsoids, then it will be unclassified (for 2D example see Fig. 4a).

4.2 K-nearest neighbourhood classifier (kNN)

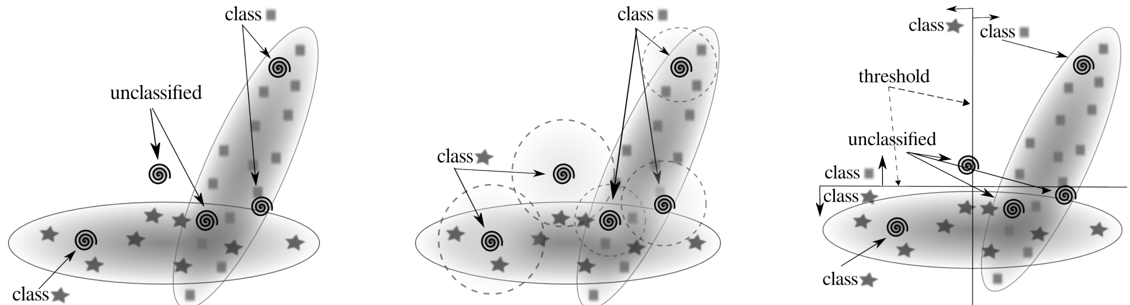
Having a labelled training dataset for each new object we create around it a virtual hyper-sphere which encloses k objects from a training dataset [3]. The radius of the hyper-sphere is given by the biggest distance between a new object and one of the k objects from training dataset. The object then is classified to this class which representatives are the majority within the k-object dataset. In this case only when in the k neighbourhood the number of different class members is equal an unclassification occurs (see Fig. 4b).

4.3 Double threshold classifier (DT)

The *double threshold* classifier can be described as a combination of threshold classifier with voting. For each object two (or more in multi-dimensional case) parallel classification by threshold classifier takes place. An object is classified to a class if all classifiers return the same result (vote similarly), otherwise it becomes unclassified (see the Fig. 4c). Such definition assures that the miss-classification of an object is on very low level, which is the advantage we are looking for. Moreover, it returns a considerable amount of unclassified objects, which can be classified once again.

5. Snowflake discriminant methods

The snowflake discriminant methods are a distinguished class of snow particle classifiers, which have one common feature. Each of these classifiers correctly



(a) Mahalanobis minimal distance (b) K-nearest neighbourhood, k=3 (c) Double threshold

Fig. 4 The comparison of classification methods for two class problem. Spirals point, the classified object; circle mark, the neighbourhood.

classifies all or almost all (more than 98%) of graupel particles, but its efficiency to classify snowflake is much lower (less than 50%). Therefore, as a general classifier their performance is unsatisfactory. Yet, when we notice that they never misclassify a graupel as a snowflake, we can exploit them for snowflake discrimination in the input data. Such an application allows to create a system, which in first step enable to recognize a small part of input data (and only snowflake particles), but simultaneously assures that this discrimination is free of error.

There have been defined following techniques:

- α flake number (number of snow particles in one image),
- β corner number (number of bending of the particle contour),
- γ hole number (number of holes in the snow particle) and flake number,
- δ hole number and corner number,
- ϵ flake number and perimeter (of particle),
- ζ flake number and corner number.

6. Proposed system data flow

Having so many snow particle features and classification methods, it is possible to design many systems. In this work we describe three systems designed for automatic snow particle classification into snowflake and graupel.

6.1 Simple classification system

The simple classification system describes each object by a pair of features and uses for classification one of the three classification techniques described in Section 4. Therefore, in our research for each feature pair and each classifier separate system was designed.

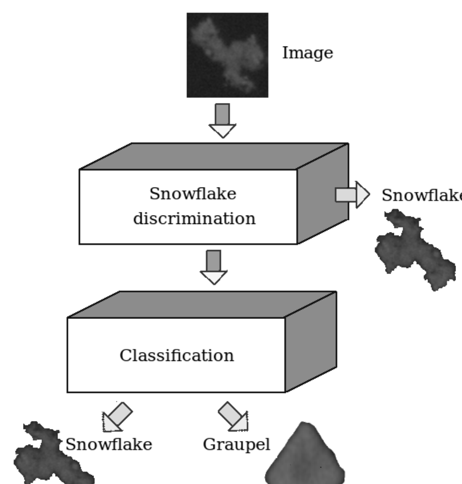


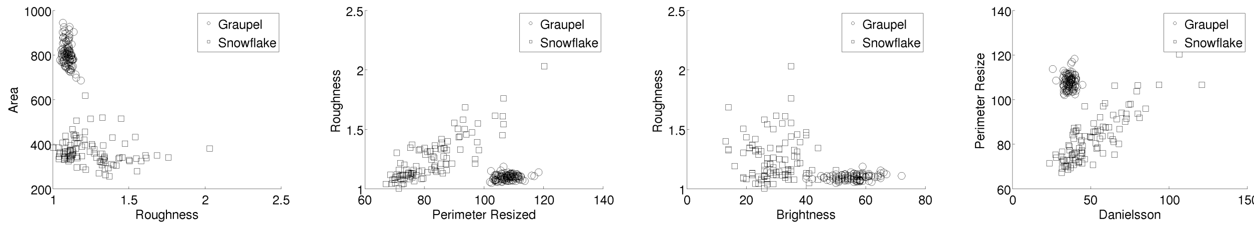
Fig. 5 The decision flow in the system with snowflake discrimination applied.

6.2 Snowflake discrimination methods for classification improvement

This is a two step classifier, which data flow is presented in Fig. 5. Firstly the snowflake discrimination threshold classifier is applied. Those particles which are not classified as snowflake are then classified by the simple classification system. This combination allows in first step to correctly classify a part of data, which possibly could be miss-classified in the second step. That should improve the results. Many systems combining each from snowflake discriminant techniques with each feature pair classified by *MMD* or *kNN* classifier were tested.

6.3 Cascading decision system

The idea of the cascading system is following. Firstly, the input data goes through the *DT* classifier. This step could be repeated for the unclassified data with



(a) H - pair (b) I - pair (c) J - pair (d) L - pair

Fig. 7 The examples of achieved scatter plots for selected feature pairs.

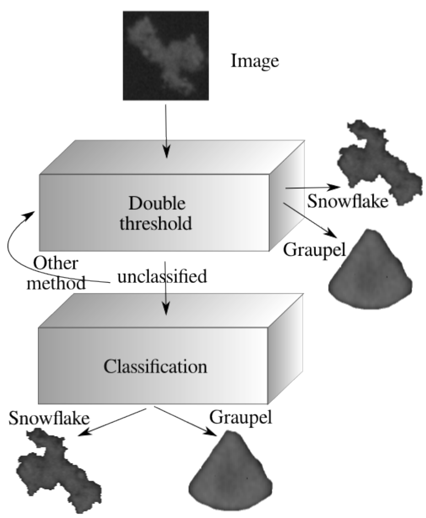


Fig. 6 The decision flow in the cascading decision system.

different set of feature pairs. The main advantage of this part of processing is the low miss-classifcation ratio with comparison to other systems. Next, the unclassified particles are classified by the statistical classifiers (see Fig. 6).

7. Result and discussion

7.1 Image database description

The snow particle database contains 8480 gray scale images. It is divided into training and testing datasets. The training dataset consists of 460 snowflake images and 461 graupel images. The testing dataset contains 3924 snowflake and 3635 graupel images.

7.2 Feature pair selection

There are many features which could be used in the automatic snow particle classification system. Therefore, firstly, we narrow the research only to those which combination should bring good results. In order to select the best ones the double feature scatter plots have been created for all combination (examples are depicted in Fig. 7). It allowed to select 25 pairs of features, but the results over 60% achieved only 19 pairs, therefore only those are presented in Table 1.

7.3 Simple classification system

Results achieved by applying the simple classification system are presented in Table 2 for *MMD* and *kNN* and Table 3 for *DT* classification. The second column of Table 2 presents the results for the *MMD* classifier. Here only three methods result worse than 70%, while two of them **N** (perimeter and volume) and **J** (brightness and roughness) have the classification ratio on the level of 80%. Three right columns of Table 2 show the results achieved for the *kNN* classifier. This method has been applied with three different values of k parameter equal to 5, 15, and 25. It is worth to notice, that increasing the number of neighbours for voting does not always improve the classification ratio. Yet, it is difficult to find any pattern of such behaviour. Generally, larger the neighbourhood, worse the average performance of all systems is. Therefore, bigger neighbourhood was not taken into consideration. The highest classification ratios have been achieved for k = 25, and there the threshold of 80% is overcome considerably. The best 3 results are for **C** (Hu moment 1st order original and roughness), **E** (Hu moment 1st order original and Danielsson), and **J** (brightness and roughness). In case of **C** and **E** classifiers we can see that if the classification ratio for one-dimentional classification was high (eg., Hu moment 1st original – 80.99%, Danielsson – 80.67% – and roughness – 84.48%) it also works well in combination with other features. On the other hand, the brightness classification ratio in one-dimensional classification was low – 75.16, what in consequence diminish the roughness – 84.48% – performance slightly in the case of **J** pair.

Table 3 presents the classification ratio achieved for *DT* classifier. In this case all classification parameters are shown: correct classification, miss-classification and unclassified ratios. This method assumes lower correct classification ratio than those previously described, but the miss-classification ratio should be much lower as well. Therefore, the results around 70% for the **B** (Hu moment 1st order original and volume), **D** (Hu moment 1st order original and area), and **F** (Hu moment 1st order original and brightness) methods are

Table 1 The selected pair combinations.

Symbol	Feature 1	Feature 2
A	Hu 1st order thresh.	area
B	Hu 1st order org.	volume
C	Hu 1st order org.	roughness
D	Hu 1st order org.	area
E	Hu 1st order org.	Danielsson
F	Hu 1st order org.	brightness
G	roughness resized	area
H	roughness	area
I	perimeter resized	roughness
J	brightness	roughness
K	Danielsson	area
L	Danielsson	perimeter resized
M	volume	area
N	volume	perimeter
O	perimeter resized	area
P	concave number	volume org.
R	concave number	area
S	brightness	area
T	brightness	concave number

Table 2 The simple system classification results. The second right column presents result for MMD classification modul, while the three left columns show results for kNN for different k values.

Method	MMD	kNN		
Parameters		k = 5	k = 15	k = 25
A	68.59	68.78	69.07	70.01
B	79.10	78.66	78.69	78.45
C	77.38	87.62	87.62	87.72
D	68.59	69.29	69.07	70.01
E	76.11	81.69	83.36	85.12
F	77.06	77.97	76.77	77.55
G	70.29	69.92	70.02	70.26
H	71.36	69.93	70.21	70.26
I	72.63	66.44	56.62	54.78
J	80.71	80.65	77.34	77.47
K	74.32	78.12	76.76	74.88
L	64.73	78.41	77.83	77.27
M	74.98	72.56	71.91	71.27
N	80.83	77.77	78.12	77.89
O	75.66	76.93	75.80	73.93
P	77.83	77.14	76.74	76.45
R	71.42	70.05	69.85	70.08
S	72.46	72.13	72.23	71.33
T	75.22	73.16	74.16	74.84

Table 3 The classification results achieved for the simple system for the DT classification method.

Method	Correct	Unclassified	Total	Missed
A	49.24	44.27	93.50	6.50
B	68.57	19.01	87.60	12.40
C	67.75	29.62	97.37	2.63
D	68.44	15.19	83.62	16.38
E	64.65	32.36	70.10	29.90
F	71.22	17.53	87.75	12.25
G	57.58	44.54	98.13	1.87
H	56.35	42.47	98.84	1.16
I	28.98	70.51	99.50	0.50
J	65.22	30.67	95.89	4.11
K	53.51	44.71	98.23	1.77
L	26.63	71.77	98.40	1.60
M	60.76	24.70	85.46	14.54
N	27.78	71.16	98.94	1.06
O	43.34	28.75	72.09	27.91
P	46.77	38.46	85.22	14.78
R	49.90	28.10	78.00	22.00
S	59.65	28.75	88.40	11.60
T	45.48	42.85	88.33	11.67

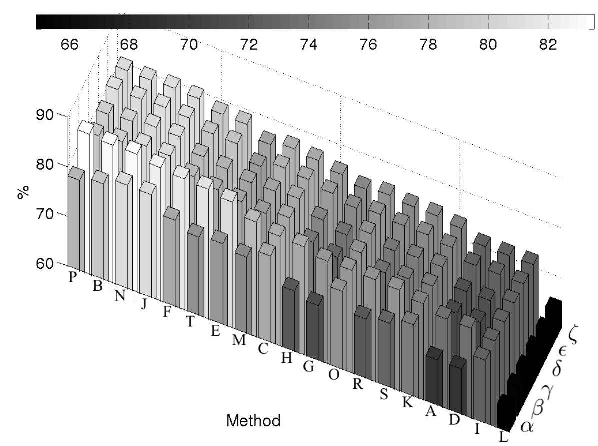


Fig. 8 Classification efficiency for the MMD classifier applied after one from six snowflake discriminant methods.

very high. Unfortunately, in the cases of high correct classification ratio the miss-classification is high as well and overcomes 10%.

It is worth to notice, that the sum of correct classification ratio with unclassified ratio (which is presented in column *Total*, Table 3) in many cases is above 90%, hence it seems to be a good idea to combine those methods with high total percentage into a cascading system.

7.4 Snowflake discrimination methods for classification improvement

The results achieved by systems exploiting the snowflake discrimination are shown in Figs. 8 and 9. In order to better visualize the differences in classification accuracy the presented techniques are sorted from those giving the best results in the left part of the plot to the worst one in the right side.

The conclusion drawn from this experiment is that the highest increase in correct classification ratio is when the β (corner numbers) snowflake discriminant method is utilized, and this pattern is visible for all tested systems. When applying one of the γ, δ, ϵ, and ζ the results also improve in comparison with the results from previous experiment, see Table 2. Only in case of α (flake number) the efficiency is worse in some cases (methods **C**, **J**, **N**). That must be influenced by wrong graupel classification by the snowflake discriminant method. The better the result the brighter colour depicts its pillar. The best result achieved in this experiment has the correct classification ratio of 94.14% when the system uses the snowflake discrimination method β and k=25 for *kNN* classifier.

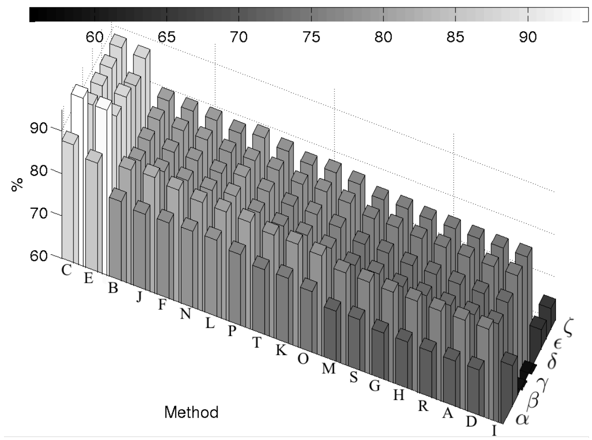


Fig. 9 Classification efficiency for the k=25 of kNN classifier applied after one from six snowflake discriminant methods.

7.5 Cascading decision system

Table 4 presents results when the cascading system is composed of 2 steps of DT classification only. As a first *DT* the **I** (roughness and perimeter resized) classifier is applied as it has the lowest wrong classification ratio of 0.5% (as shown in previous experiment, please refer to Table 3). Different results have been achieved for different feature pairs applied in the second step of *DT*. The selected methods characterized small wrong classification error less than 10%. For comparison Table 5 presents results when in the second step the *kNN* (k = 25) or *MMD* classifier is used. In case of this system there were chosen those methods, which correct classification ratio was higher than 77% (it is a threshold above which the best five efficiency ratios are found).

From that experiment we can see that the classification as well as the miss-classification ratio when exploiting only two *DT* classifiers are still low. On the other hand, the combination of *DT* classifier with *kNN* or *MMD* resulted in the classification ratio around 80%.

In the next experiment we have created system in which firstly two *DT* classifiers have been used, and after them followed the *kNN* or *MMD* classifiers. There were selected two method combination for *DT* method. One was when **I** pair is followed by **H** pair, as this combination has the smallest wrong classification error. Results achieved for this systems are presented in Table 6. The second pair was **I** pair followed by **J** pair, this combination has the highest correct classification ratio. Results for this cascading system for this classifier are presented in Table 7. The achieved performance is better in case when we apply two passes through *DT* classifier and follow it with statistical classifiers. This experiments shows that combining many different approaches for feature classification improve the results. Moreover, here we can also notice that applying the *kNN* classifier brings better results than *MMD* classifier.

8. Conclusion

Three approaches have been presented for multi-feature

Table 4 Results achieved for two iteration of DT cascading system.

Method	Classification	
in 2nd DT	Correct	Missed
J	**68.50**	**4.34**
C	61.64	2.73
G	58.63	1.77
K	57.35	1.85
H	**56.67**	**1.20**
A	56.17	6.43
H	55.34	5.7
N	46.73	1.47
L	30.27	1.76

Table 5 Results achieved for cascading system when as a first was used the DT classifier for method **I** followed by k=25 of kNN classifier (left) or MMD classifier (right).

	Correct classification	
Method	k = 25 NN	MMD
B	81.12	81.53
C	88.37	78.32
E	87.30	–
J	80.38	83.01
L	78.02	–
N	80.43	83.36
P	–	79.56
Y	79.96	80.02

Table 6 Results achieved for cascading system when as a first was used the DT classifier for method **I** followed by DT method **H** and finally followed by k=25 of kNN classifier (left) or MMD (right).

	Correct classification	
Method	k = 25 NN	MMD
B	85.04	85.00
C	88.38	81.45
E	87.66	–
J	84.44	86.73
L	79.36	–
N	85.04	86.69
P	–	83.13
Y	83.75	84.27

Table 7 Results achieved for cascading system when as a first was used the DT classifier for method **I** followed by DT method **J** and finally followed by k=25 of kNN classifier (left) or MMD (right).

	Correct classification	
Method	k = 25 NN	MMD
B	83.56	82.91
C	87.93	84.56
E	87.64	–
J	81.94	84.03
L	86.45	–
N	83.62	84.27
P	–	82.39
Y	82.26	81.58

snow particle classification into snowflakes and graupels. All of them as a mean for the decision making take a pair of features, which have been selected from broad range of features designed for this problem classification.

Firstly, the simple, one step system is presented, where the input data flows through the classification module only. There are compared 19 feature pairs and the performance of *MMD* classifier, *kNN* classifier with newly introduced *DT* classifier. The best results achieved for this system was 87%. This system due to its simplicity saves classification computation time with little los of accuracy.

Secondly, the influence of applying the snowflake discrimination method and applying the simple system only for unclassified images is discussed. Here also all 19 pairs are taken into consideration, but the classification is done by the *MMD* or *kNN* classifiers. The experiment shows that enriching the system with this one step can improve the results slightly almost in all cases. Additionally, the correct classification ratio is over 90% when β snowflake discrimination technique is applied before the **C** or **E** feature pairs classification with *kNN*. This technique combination allowed to improve system classification accuracy.

Finally, the cascading decision system has been introduced. In this case the *DT* classifier is combined with *MMD* or *kNN* classifier. It shows that there also could be achieved satisfactory results, however the maximal performance was around 88%. Sometimes the need of avoiding wrong classification is more important than the classification accuracy, what can be achieved by this system. For example, when we consider to estimate the relationship between radar reflectivity and snowfall rate, we may permit lower particle classification ratio, but the misclassification is inadmissible [16].

Generally, the presented systems for snow particle classification, based on the feature selection from images, create a powerful tool. Their possible applications are various. For instance, the imaging system with a specialized software could be useful in fields like radar meteorology or in research concerning the physics of precipitation.

References

[1] V. C. Chen. "Evaluation of Bayes, ICA, PCA and SVM methods for classification", RTO SET Symposium on Target Identification and Recognition Using RF Systems, vol.37, pp.1–12, October 2004.

[2] A. El-Magd, V. Chandrasekar, V. N. Bringi, and W. Strapp, "Multiparameter radar and in situ aircraft observation of graupel and hail", IEEE Trans. GeoSci. Remote Sens., vol.38, no.1, pp.570-578, 2000.

[3] E. Gose, R. Johsonbaugh, and S. Jost, Pattern Recognition and Image Analysis, Prentice-Hall, Inc., Upper Saddle River, NJ, 1996.

[4] T. Harimaya,"The relationship between graupel formation and meteorological conditions", J. Meteor. Soc. Jap., vol.66, no.4, pp.599-606, 1988.

[5] T. Harimaya and Y. Nakai, "Riming growth process contributing to the formation of snowfall in orographic areas of Japan facing the Japan sea", J. Meteor. Soc. Japan., vol.77, no.1, pp.101-115, 1999.

[6] T. Harimaya, H. Ishida, and K. Muramoto, "Characteristics of snowflake size distributions connected with the difference of formation mechanism", J. Meteor. Soc. Japan, vol.78, no.3, pp.233-240, 2000.

[7] T. Harimaya, H. Kodama, and K. Muramoto, "Regional differences in snowflake size distributions", J. Meteor. Soc. Japan, vol.82, no.3, pp.895-903, 2004.

[8] M-K. Hu, "Visual pattern recognition by moment invariants", IRE Trans. on Information Theory, IT-8, pp.179-187, 1962.

[9] J.S. Marshall and K.L.S. Gunn, "Measurements of snow parameters by radar", J. Atmos. Sci., vol.9, pp.322–327, 1952.

[10] H. Mizuno, "Statistical characteristics of graupel precipitation over the Japan Islands", J. Meteor. Soc. Japan., vol.70, no.1, pp.115-121, 1992.

[11] J. Myung, "Tutorial on maximum likelihood estimation", J. Math. Psychol., vol.47, pp.90–100, 2003.

[12] W.K.Pratt, Digital Image Processing: PIKS Inside, 3rd Edition, John Wiley & Sons, Inc., 2001.

[13] J. C. Russ, The Image Processing Handbook, 2nd edition, CRC Press, Springer, and IEEE Press, 1998.

[14] H. Servomaa, K. Muramoto, and T. Shiina, "Snowfall characteristics observed by weather radars, an optical lidar and a video camera", IEICE Trans. Inf. & Syst., vol.E85-D, no.8, pp.1314-1324, 2002.

[15] K. Seto, K. Nurzynska, M. Kubo, K. Muramoto, and T. Shiina, "Classification of snow particles using image processing", IEICE Technical Report, (Japanese), PRMU2009-98, pp.21-26, November 2009.

[16] H. Yamada, H. Uyeda, K. Kikuchi, M. Maki, K. Iwanami, "Dual-Doppler radar observations on factors causing differences in the structure of snow clouds during winter monsoon surges", J. Meteor. Soc. Japan. vol.82, no.1, pp.179-206, 2004.

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